Chair of Software Modeling and Verification (Computer Science 2) | RWTH AACHEN UNIVERSITY

Master Thesis at the
Software Modeling and Verification Group (MOVES)

Your Awesome Thesis Title

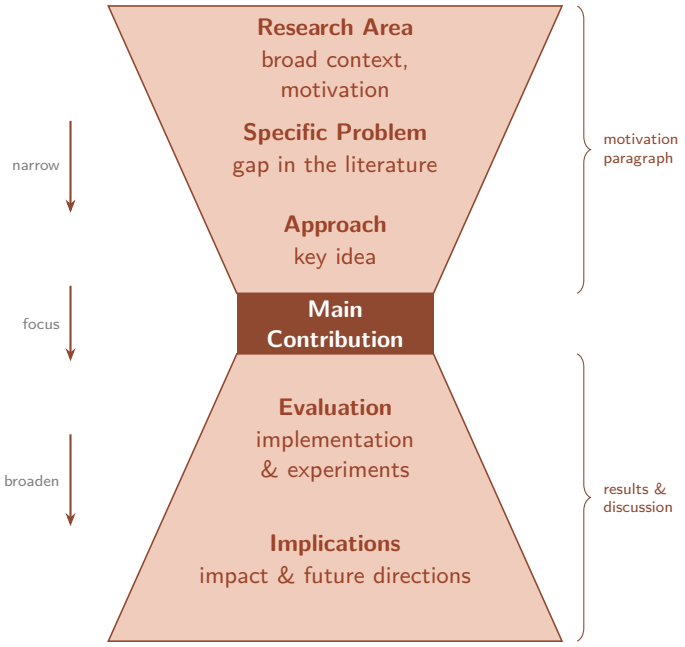
by

Your Name

This written work was submitted to
Lehrstuhl für Informatik 2, RWTH Aachen University.

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Abstract



Hourglass structure of a good abstract. Start broad (research area and problem), funnel down to the specific contribution, then widen again to state results and implications.

Write a self-contained summary of **roughly 150–250 words** following the hourglass structure above. Cover the following six points in order:

1. **Research area.** Name the field (e.g., strategy synthesis, probabilistic program verification, operational semantics) and state in one sentence what the field studies and why it matters.
2. **Specific problem.** State precisely which problem this thesis addresses and why it is challenging or not yet fully solved by existing approaches.
3. **Approach.** Describe the key idea of your contribution in one or two sentences. Avoid mathematical notation and abbreviations, and use easily understandable language.
4. **Theoretical results.** Name the main property proven (soundness, completeness, an equivalence result, or a complexity bound) and state it informally.

5. **Implementation and evaluation (if applicable).** Name the tool and the benchmark set and give the headline result: how many more examples are solved compared to the baseline and what is the runtime overhead.
6. **Implications.** Explain how your results solve the issue or fill the gap you pointed out beforehand.

Acknowledgements

I want to express my most profound appreciation to my supervisor! I might want to thank someone else as well ... Many thanks also to Jan-Christoph Kassing for letting me adapt his thesis template and take over its writing guidelines [Kas26]. The tips and tutorials from this template would not be here in such detail without his help.

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Chapter 1

Introduction

The introduction motivates the topic and provides an overview of the thesis. It can be seen as a longer version of the abstract that follows the hourglass structure again, but with emphasis on the top part of the hourglass. However, you can now use citations and explain everything in more detail. It should be accessible to any computer scientist without prior knowledge of the specific area.

The background and motivation part should cover the following points:

1. Name the research area and describe informally what objects are studied (e.g., Markov automata, probabilistic programs, POMDPs) and what property is of interest (e.g., reachability, complexity, safety).

2. Explain why the property matters.

3. Give a small, self-contained example that illustrates the problem informally.

4. Explain why answering the question automatically is non-trivial and what makes the problem hard in general.

The open-problem part should contain:

1. A summary of what is already known in the area (prior results, existing automated techniques, tool support).

2. A precise identification of the gap: e.g., no automated procedure exists, existing techniques are unsound or incomplete for a relevant subclass, or no efficient algorithm is known.

3. A clear statement of what this thesis aims to achieve.

Related Work

Discuss the most relevant prior work (either here, or in a separate chapter depending on the scope of existing work). For each cited work, address the following:

1. What does the work do? Describe the approach in one to three sentences and do not just list references.
2. How does it relate to this thesis? Does it solve a special case, use a different technique, or target a different formalism?
3. What limitation motivates your own contribution?

Contribution

List one item per concrete contribution. Each item should:

1. Identify what is new (a technique, a criterion, a framework, an implementation, or an experimental evaluation).
2. State the proven property, e.g., soundness, completeness, or a complexity bound.
3. Reference the theorem or section in which the contribution appears.

Structure

End the introduction by outlining the remaining chapters. Give at least one sentence per chapter describing its content. Do not simply write "Chapter X presents the main results."

Example: "The remainder of this thesis is organized as follows. Chapter 2 introduces the formal background required throughout this thesis, including ... Then, Chapter 3 presents ... Chapter 4 describes the implementation and the experimental evaluation. Finally, Chapter 5 summarizes the results and discusses directions for future work."

Chapter 2

Preliminaries

The opening paragraph of a chapter should briefly outline the chapter structure by naming each section and stating what it covers.

In the preliminaries you can name the studied formalism (e.g., model checking, probabilistic programs, parameter synthesis) and cite standard references for further reading (e.g., [BK08]). Do not add a citation to every definition.

2.1 Basic Definitions

Introduce the formal objects of the theory bottom-up: start with the most elementary notion (e.g., transition systems, Markov chains) and build towards the property of interest (e.g., LTL properties, complexity).

Each definition in this section should be accompanied by:

1. A sentence stating what the definition formalizes and why it is needed.
2. The formal statement in a Definition box.
3. A brief explanation of each component of the definition.
4. At least one example.

Cover the following in order:

1. **Syntactic Objects.** Define the elementary building blocks of the formalism (e.g., lattices, Markov models, program graphs), relationships between the individual objects and relevant formal properties of interest.
2. **Derived Notions.** Define derived concepts such as strategies, reachability, bisimulations, etc.

3. **The Property of Interest.** Give the formal definition of the property the thesis analyzes (e.g., termination, equivalence, strategy synthesis, almost-sure termination).

How to Reference Definitions, Theorems, and Examples

Never write numbers such as "Definition 2.1" by hand. Instead, put a `\label` directly after `\begin{...}` (for figures and tables: after `\caption`). Then reference it with `\Cref` at the beginning of a sentence and with `\cref` everywhere else. The reference prints the type and the number as one link, e.g., `\cref{thm:absorption}` prints "Theorem 3.6" and `\cref{def:dtmc,ex:dtmc}` prints "Definition 3.1 and Example 3.2". For expressive reference pointers, start every label with the prefix of its type (see Table 2.1).

Table 2.1: Label prefixes.

Type	Prefix	Example
Chapter	`ch:`	`\label{ch:preliminaries}`
Section, subsection	`sec:`	`\label{sec:known-results}`
Appendix chapter	`app:`	`\label{app:omitted-proofs}`
Definition	`def:`	`\label{def:dtmc}`
Lemma	`lem:`	`\label{lem:aux}`
Theorem	`thm:`	`\label{thm:absorption}`
Corollary	`cor:`	`\label{cor:key}`
Proposition	`prop:`	`\label{prop:key}`
Observation	`obs:`	`\label{obs:simple-fact}`
Remark	`rem:`	`\label{rem:scope}`
Example	`ex:`	`\label{ex:dtmc}`
Assumption	`asm:`	`\label{asm:finite-states}`
Equation	`eq:`	`\label{eq:fixpoint}`
Figure	`fig:`	`\label{fig:architecture}`
Table	`tab:`	`\label{tab:results}`
Algorithm	`alg:`	`\label{alg:value-iteration}`

2.2 Known Results from the Literature

Present the lemmas and theorems from the literature that are used in Chapter 3. You do not have to prove these results unless the proof is insightful for the rest of the

thesis. Always cite the original source. Each result in this section should be presented as follows:

1. A sentence explaining why the result is important and how it will be used in the thesis.
2. The formal statement in a Lemma, Proposition, or Theorem box, with a citation in the title.
3. A proof indication: either a short proof, "Proof: see [DMRN24]", or a forward reference to the appendix.
4. (Optional) A brief Remark box on the scope or limitations of the result if that is relevant for understanding its use in Chapter 3.

This section collects known results from the literature that are used in the rest of the thesis. Organize them as follows:

1. **Auxiliary lemmas.** State small technical results about the basic objects from Section 2.1 (e.g., equivalence relations, closure conditions, or monotonicity). These are typically needed in the proofs of Chapter 3.
2. **Characterization theorem(s).** State the main theoretical results from the literature about the property of interest. These serve as a reference point for understanding what is already known.
3. **Existing automated technique(s).** If the thesis extends an existing automated method (e.g., loop-invariant computation, value iteration, parameter synthesis), state its correctness theorem here.

The following boxes show all environments of the template and when to use them.

Theorem 2.1 (Name of the Key Theorem)
The most important results.

Theorem 2.2 (Name of a Theorem, [DMRN24, Theorem 6.7])
A result from the literature: cite the source with the exact location in the title.

Proposition 2.3 (Name of a Proposition)
A result of medium importance, e.g., a self-contained statement that is not the main result.

Lemma 2.4 (Name of an Auxiliary Lemma)
An auxiliary result used to prove other results.

Corollary 2.5 (Name of a Direct Consequence)
A direct consequence of another result.

Observation 2.6
A simple fact that follows almost directly from the definitions, with a short proof.

Proof 1
Short proof of the observation. □

Remark 2.7
A comment on a definition or result, e.g., why an assumption is needed or how it relates to the literature.

Assumption 1:
A condition that is required throughout the thesis, e.g., that all models are finite.

Definition 2.8 (Name of the Defined Notion, [DMRN24, Definition 4.2])
A definition (if taken or adapted from another source, cite it in the heading).

Example 2.9
An example.

You can use colors to highlight differences in boxes or even difficult parts in proofs. However, do not make the colors too distracting and everything has to be readable in black and white as well.

Chapter 3

[Main Contribution]

This chapter contains the main theoretical (and possibly technical) contribution of the thesis. It should present a coherent body of new results, organized into sections.

For a theory-focused thesis: introduce new notions, state lemmas/theorems, provide proofs, and illustrate results with examples. For an implementation-focused thesis: additionally describe the algorithm, key design decisions, and data structures.

The opening paragraph of this chapter should cover the following points:

1. State what new objects, criteria, or techniques this chapter introduces.
2. Introduce a running example that will be used throughout to illustrate each new definition and result.
3. Outline the chapter structure by referencing the main sections.

3.1 [Central Definitions / New Framework]

For self-introduced notions it is very important to explain every part clearly and use multiple examples to illustrate everything. Use one running example (e.g., Example 3.2) throughout the whole section and maybe some additional small examples to highlight special cases.

3.2 [Main Results]

Each result in this section should be presented as follows:

1. State and prove all auxiliary lemmas before the main theorem.
2. State the main theorem. Typical statements are:
    - **Soundness:** If the technique returns YES, then the property holds.

- **Completeness:** If the property holds, the technique returns Yes.
- **Equivalence:** The new criterion holds if and only if an existing criterion holds.

3. Provide a complete proof, structured into numbered cases for long arguments. Reference definitions and earlier results by label. If a proof is long or very technical, give a proof sketch here and move the full proof to the appendix (see Appendix A).
4. Apply the theorem to the running example, walking through every step explicitly.

Section 3.3 shows all of this in a short example.

3.3 Section Example: Absorption in Markov Chains

TODO: Add a motivating example.

We now show that a random system that can always reach a final state eventually ends up there.

A discrete-time Markov chain moves between states in discrete steps, where the next state is chosen randomly and only depends on the current state. We use a simplified version of the definition in [BK08, Definition 10.1].

Definition 3.1 (Discrete-Time Markov Chain)
A *discrete-time Markov chain* (DTMC) is a pair $\mathcal{D} = (S, \mathbf{P})$ of a finite, non-empty set of *states* S and a *transition probability function* $\mathbf{P} \colon S \times S \to [0, 1]$, where $\mathbf{P}(s, t)$ is the transition probability from $s \in S$ to $t \in S$, and $\sum_{t \in S} \mathbf{P}(s, t) = 1$ for all $s \in S$.

Example 3.2
Let $S = \{s_0, s_1, s_2\}$, $\mathbf{P}(s_0, s_0) = \mathbf{P}(s_0, s_1) = 1/2$, $\mathbf{P}(s_1, s_0) = \mathbf{P}(s_1, s_2) = 1/2$, $\mathbf{P}(s_2, s_2) = 1$, and all other transition probabilities 0. $\mathcal{D}_{ex} = (S, \mathbf{P})$ is a DTMC, since S is finite and the transition probabilities of every state sum up to 1.

To reason about several steps, we extend $\mathbf{P}$ to n steps.

Definition 3.3 (Multi-Step Probabilities)
Let $\mathcal{D} = (S, \mathbf{P})$ be a DTMC. For $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$ and $s, t \in S$, let $\mathbf{P}^0(s, t) = 1$ if $s = t$, $\mathbf{P}^0(s, t) = 0$ otherwise, and $\mathbf{P}^{n+1}(s, t) = \sum_{u \in S} \mathbf{P}^n(s, u) \cdot \mathbf{P}(u, t)$.

Definition 3.4 (Absorbing States, [BK08, Notation 10.6])
Let $\mathcal{D} = (S, \mathbf{P})$ be a DTMC. A state $s \in S$ is *absorbing* if $\mathbf{P}(s, s) = 1$.

Generally, $\mathbf{P}^n(s, t)$ is the probability to be in state t after exactly n steps when starting

in state s [BK08, p. 768], and a state is absorbing if and only if it cannot be left. In Example 3.2, state s_2 is absorbing and we have $\mathbf{P}^2(s_0, s_2) = \sum_{u \in S} \mathbf{P}(s_0, u) \cdot \mathbf{P}(u, s_2) = \mathbf{P}(s_0, s_1) \cdot \mathbf{P}(s_1, s_2) = 1/4$.

We first show that the multi-step probabilities again form a probability distribution, which we need in the proof of Theorem 3.6.

Feedback: Careful with forward references, as they might break reading flow.

Lemma 3.5 (Multi-Step Probabilities are Distributions)
Let $\mathcal{D} = (S, \mathbf{P})$ be a DTMC. For all $n \in \mathbb{N}$ and $s \in S$, we have $\mathbf{P}^n(s, t) \geq 0$ for all $t \in S$ and $\sum_{t \in S} \mathbf{P}^n(s, t) = 1$.

Proof 2
We prove the claim by induction on n.

Base case ($n = 0$). By Definition 3.3, we have $\mathbf{P}^0(s, s) = 1$ and $\mathbf{P}^0(s, t) = 0$ for all $t \neq s$. Hence, all values are non-negative and $\sum_{t \in S} \mathbf{P}^0(s, t) = \mathbf{P}^0(s, s) = 1$.

Induction step ($n \to n + 1$). As induction hypothesis (IH), assume that for every $s \in S$, the values $\mathbf{P}^n(s, u)$ with $u \in S$ are non-negative and sum up to 1, i.e., Lemma 3.5 holds for n. Let $s, t \in S$. By Definition 3.3, we have

$$\mathbf{P}^{n+1}(s, t) = \sum_{u \in S} \mathbf{P}^n(s, u) \cdot \mathbf{P}(u, t).$$

Every summand is the product of $\mathbf{P}^n(s, u) \geq 0$ (by the IH) and $\mathbf{P}(u, t) \geq 0$ (by Definition 3.1), so $\mathbf{P}^{n+1}(s, t) \geq 0$. Moreover, we have

$$\begin{aligned} \sum_{t \in S} \mathbf{P}^{n+1}(s, t) &= \sum_{t \in S} \sum_{u \in S} \mathbf{P}^n(s, u) \cdot \mathbf{P}(u, t) = \sum_{u \in S} \mathbf{P}^n(s, u) \cdot \sum_{t \in S} \mathbf{P}(u, t) \\ &= \sum_{u \in S} \mathbf{P}^n(s, u) \overset{IH}{=} 1. \end{aligned}$$

The first equality holds by Definition 3.3. The second equality swaps the two finite sums and moves $\mathbf{P}^n(s, u)$, which does not depend on t, out of the inner sum. The third equality holds since $\sum_{t \in S} \mathbf{P}(u, t) = 1$ for all $u \in S$ by Definition 3.1. □

Our main result states that a DTMC eventually ends up in an absorbing state $s_\perp$ with probability one if this state can be reached from every state. It is a variant of the theorem on almost-sure reachability in [BK08, Theorem 10.29], which differs from ours in three points:

1. it extends to a set of absorbing states instead of a single absorbing state $s_\perp$.
2. it only requires that this set can be reached from every state that is reachable from the initial state s, and it shows an equivalence. We require that $s_\perp$ can be reached from every state, which simplifies the proof, and only show one direction.

3. it states the conclusion as the probability of all paths from s that eventually reach this set, which needs a probability measure on paths. Since $s_\perp$ cannot be left, $\mathbf{P}^n(s, s_\perp)$ is the probability of reaching $s_\perp$ within n steps, so we can use $\lim_{n\to\infty} \mathbf{P}^n(s, s_\perp)$ instead.

Theorem 3.6 (Absorption, cf. [BK08, Theorem 10.29])
Let $\mathcal{D} = (S, \mathbf{P})$ be a DTMC and let $s_\perp \in S$ be an absorbing state that is reachable from every $s \in S$, i.e., there exists an $n \in \mathbb{N}$ with $\mathbf{P}^n(s, s_\perp) > 0$. Then $\lim_{n\to\infty} \mathbf{P}^n(s, s_\perp) = 1$ for all $s \in S$.

Proof 3 (Sketch)
Since S is finite, there exist $m \in \mathbb{N}$ and $\delta > 0$ such that from every state, the DTMC is in $s_\perp$ after m steps with probability at least δ. As $s_\perp$ is absorbing, the probability of not being in $s_\perp$ after $k \cdot m$ steps is thus at most $(1-\delta)^k$, which tends to 0 for $k \to \infty$. The full proof can be found in Appendix A.1. □

Example 3.7
In Example 3.2, the absorbing state s_2 can be reached with positive probability from s_2 in zero steps, from s_1 in one step, and from s_0 in two steps. Hence, by Theorem 3.6, the DTMC ends up in s_2 with probability one, no matter in which state it starts.

Feedback: Explain what happens if s_2 cannot be reached from some state.

Chapter 4

Evaluation

This chapter demonstrates that the approach from Chapter 3 works in practice.

A good evaluation chapter:

- Describes clearly what was implemented and provides an accessible link to the tool/repository (technical details – setup, features, usage tutorial – can be deferred into a clear readme there).
- Clearly states where the application examples/benchmarks are taken from.
- Presents results honestly, including cases where the technique does not help.
- Interprets the numbers (runtime, results, failing examples) and connects them back to the theory.

4.1 Implementation (optional)

The implementation section should cover:

1. The name of the tool implemented (or extended), a brief description of its overall architecture (e.g., the pipeline from input model to verification results), and any external solvers/tools used.
2. If you extended an existing tool, which module(s) you added or changed and how they fit into the existing architecture.

4.2 Experimental Setup

The experimental setup should specify:

1. **Benchmark set.** Which set was used (and why)? How many examples total? Are there relevant subsets by category, strategy, or problem type?

2. **Configurations compared.** At minimum include the unmodified baseline and the tool with your extension. List any competing tools with a brief description.
3. **Hardware and time limits.** CPU model, amount of RAM, operating system, and per-example wall-clock timeout.

4.3 Results

The results section should:

1. Be clearly readable, preferably in a table (see Table 4.1) (and/or graph).
2. Summarize the headline finding directly below the table, citing concrete numbers (additional YES answers, percentage improvement, overhead in seconds).
3. Point out interesting subsets or individual examples where the technique is especially effective and where the technique is not effective.
4. Highlight any surprising results to be discussed in Section 4.4.

Table 4.1 summarizes the results of our experiments.

Table 4.1: Experimental results on [*benchmark set name*]. *YES*/*NO* denotes the number of examples for which [*the property / its negation*] was proved. *MAYBE* means the tool returned no answer. *TO* means the timeout was exceeded. *Time* is the total wall-clock time in seconds.

Configuration	YES	NO	MAYBE	TO	Time (s)
Tool without extension	–	–	–	–	–
Tool with extension	–	–	–	–	–
Other tool	–	–	–	–	–

4.4 Discussion

The discussion should address the following questions:

1. Do the results match theoretical expectations from Chapter 3? If not, explain why.
2. On which types of examples does the technique help most? Identify structural properties of the benchmark examples that explain the gain.
3. Are there cases where the technique does not help or is slower? Explain the reason and relate it to the theoretical properties.

4. What are the main limitations and how could they be overcome? (This can bridge to the future work discussion in Chapter 5)

Chapter 5

Conclusion

The conclusion summarizes the thesis and discusses future work. Do not introduce any new results here. A reader who reads only the introduction and the conclusion should get a coherent picture of what was done and what remains open. It should follow the same structure as the introduction and the abstract, but with more emphasis on the bottom part of the hourglass.

The summary should address the following points:

1. Restate the problem studied and explain why it is relevant.
2. Describe the main contribution and the approach taken (reference Chapter 3).
3. Name the central theoretical result (reference Theorem 3.6) and state its guarantees informally.
4. Summarize the experimental findings – number of benchmarks solved (wrt. existing work), runtime overhead – if an evaluation was conducted (reference Chapter 4).

Each future work item should:

1. Identify a specific limitation of the current approach or a natural extension that was beyond the scope of this thesis.
2. Describe what would be needed to address it (new theoretical machinery, a different encoding, a richer formalism, etc.).
3. Explain what benefit this would bring (more examples solved, stronger guarantees, applicability to a wider class of systems).

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Appendix A

Omitted Proofs

This appendix contains the full proofs that were omitted from the main text. For every proof that you move here, follow this pattern:

1. In the main text, directly after the statement, give a proof sketch, i.e., a `proof` environment with the optional argument `[Sketch]`. It explains the main idea in a few sentences and refers to the full proof, e.g., "The full proof can be found in Appendix A.1." See the proof sketch of Theorem 3.6 for an example.
2. Here, start a new section for each proof and give its `proof` environment the optional argument `[of \cref{...}]`, so that the reader can jump back to the statement. Briefly recall the statement, so that the reader does not have to.
3. Keep the order of the main text.

A.1 Proof of the Absorption Theorem

Proof 4 (of Theorem 3.6)
Recall that $s_\perp$ is an absorbing state that can be reached from every state, and that we have to show $\lim_{n\to\infty} \mathbf{P}^n(s, s_\perp) = 1$ for all $s \in S$. For $n \in \mathbb{N}$ and $s \in S$, let

$$q_n(s) = 1 - \mathbf{P}^n(s, s_\perp) = \sum_{t \in S \setminus \{s_\perp\}} \mathbf{P}^n(s, t)$$

be the probability of not being in $s_\perp$ after n steps, where the second equality holds by Lemma 3.5. By the same lemma, we have $0 \leq q_n(s) \leq 1$. So it suffices to show that $q_n(s)$ tends to 0 for $n \to \infty$. We proceed in four steps.

Step 1: $q_n(s)$ does not increase with n. All summands in Definition 3.3 are non-negative by Lemma 3.5. Keeping only the summand for $u = s_\perp$ and using

that $s_\perp$ is absorbing, we obtain

$$\mathbf{P}^{n+1}(s, s_\perp) = \sum_{u \in S} \mathbf{P}^n(s, u) \cdot \mathbf{P}(u, s_\perp) \geq \mathbf{P}^n(s, s_\perp) \cdot \mathbf{P}(s_\perp, s_\perp) = \mathbf{P}^n(s, s_\perp).$$

Hence, $q_{n+1}(s) \leq q_n(s)$ for all $n \in \mathbb{N}$ and $s \in S$.

Step 2: A uniform bound after m steps. We find a number of steps $m \geq 1$ and a probability $\delta > 0$ such that from every state, the DTMC is in $s_\perp$ after m steps with probability at least δ.

By assumption, for every $s \in S$ there is an $n_s \in \mathbb{N}$ with $\mathbf{P}^{n_s}(s, s_\perp) > 0$. Since S is finite and non-empty, the maximum of these numbers exists, and we let $m = 1 + \max_{s \in S} n_s$, so that $m \geq 1$. As $m \geq n_s$, Step 1 yields $\mathbf{P}^m(s, s_\perp) \geq \mathbf{P}^{n_s}(s, s_\perp) > 0$ for every $s \in S$. Let $\delta = \min_{s \in S} \mathbf{P}^m(s, s_\perp)$ be the smallest such reachability probability after exactly m steps. Then $\delta > 0$, and $\delta \leq 1$ by Lemma 3.5. Since $\mathbf{P}^m(s, s_\perp) \geq \delta$ for every $s \in S$,

$$q_m(s) = 1 - \mathbf{P}^m(s, s_\perp) \leq 1 - \delta \quad \text{for all } s \in S.$$

So from every state, the probability of *not* being in $s_\perp$ after m steps is at most $1 - \delta$. For $s_\perp$ itself, this probability is even 0, as $\mathbf{P}^0(s_\perp, s_\perp) = 1$, Step 1 yields $\mathbf{P}^m(s_\perp, s_\perp) \geq 1$, so $q_m(s_\perp) = 0$.

Step 3: $q_{km}(s) \leq (1 - \delta)^k$ for all $k \in \mathbb{N}$ and $s \in S$. Intuitively, each block of m steps misses $s_\perp$ with probability at most $1 - \delta$, so k blocks miss it with probability at most $(1 - \delta)^k$. We prove the claim by induction on k.

Base case ($k = 0$). For all $s \in S$, $q_0(s) \leq 1$, since $0 \leq q_n(s) \leq 1$ for all $n \in \mathbb{N}$.

Induction step ($k \to k + 1$). As induction hypothesis (IH), assume that $q_{km}(s) \leq (1 - \delta)^k$ for all $s \in S$. Let $s \in S$. Being in t after $(k + 1) \cdot m$ steps means being in some state u after $k \cdot m$ steps and then moving from u to t in m steps. Formally, $\mathbf{P}^n$ is the n-th power of $\mathbf{P}$ viewed as an $S \times S$ matrix by Definition 3.3, so associativity of matrix multiplication yields the decomposition

$$\mathbf{P}^{(k+1)m}(s, t) = \sum_{u \in S} \mathbf{P}^{km}(s, u) \cdot \mathbf{P}^m(u, t) \quad \text{for all } t \in S.$$

We derive the following bounds:

$$\begin{aligned}
q_{(k+1)m}(s) &= \sum_{t\in S\setminus\{s_\perp\}} \sum_{u\in S} \mathbf{P}^{km}(s,u)\cdot \mathbf{P}^{m}(u,t) && \text{(decomposition)}\\
&= \sum_{u\in S} \mathbf{P}^{km}(s,u)\cdot q_m(u) && \text{(swap the sums)}\\
&= \sum_{u\in S\setminus\{s_\perp\}} \mathbf{P}^{km}(s,u)\cdot q_m(u) && (q_m(s_\perp)=0)\\
&\le \sum_{u\in S\setminus\{s_\perp\}} \mathbf{P}^{km}(s,u)\cdot (1-\delta) && (q_m(u)\le 1-\delta)\\
&= q_{km}(s)\cdot(1-\delta) && \text{(definition of } q)\\
&\le (1-\delta)^{k+1}. && \text{(IH, } 1-\delta\ge 0)
\end{aligned}$$

The bounds $q_m(s_\perp)=0$ and $q_m(u)\le 1-\delta$ hold by Step 2, and the inequality uses $\mathbf{P}^{km}(s,u)\ge 0$ (Lemma 3.5).

Step 4: $q_n(s)$ tends to 0. For $n\in\mathbb{N}$, let $k=\lfloor n/m\rfloor$. Then $k\cdot m\le n$, so Steps 1 and 3 yield $0\le q_n(s)\le q_{km}(s)\le (1-\delta)^k$ for any $s\in S$. Since $0\le 1-\delta<1$ and $k\to\infty$ for $n\to\infty$, the upper bound on the right-hand side tends to 0. Hence, $q_n(s)$, which lies between 0 and this upper bound, also tends to 0, i.e., $\lim_{n\to\infty} q_n(s)=0$. Thus, for all $s\in S$, we have

$$\lim_{n\to\infty} \mathbf{P}^n(s,s_\perp) = \lim_{n\to\infty} \big(1-q_n(s)\big) = 1-0=1. \qquad \square$$

Appendix B

Bibliography Style

This appendix explains how to write the entries of `literature.bib`. Delete it before you submit your thesis.

The style `alpha2` labels every entry by the initials of its authors and its year, e.g., [BK08], sorts the bibliography by these labels, keeps the capitalization of the `.bib` file, and prints DOIs as links. Only cited entries appear in the bibliography. Every example below is cited, so you can compare its source in `literature.bib` with the result in the bibliography.

Authors and Titles

Authors. List all authors with their full first names in the form `Last, First`, separated by `and`. Write accents in LaTeX notation, e.g., `Math{\'e}matique` in the journal of [Hal70]. Put name particles and hyphenated surnames in braces, e.g., `{von Beethoven}, Ludwig` and `{Sk\l{}odowska-Curie}, Marie`.

Titles. Write titles in Title Case exactly as they should be printed, since `alpha2` does not change their capitalization [GS90].

Entry Types

Venues. Define recurring conferences, journals, and series once with `@string` at the top of `literature.bib` and use them without braces. For conference papers, append the year with `#`, e.g., `booktitle = cav # "~'17"` prints "Proc. CAV '17". Then the same venue is always printed in the same way. Omit publishers, editors, and locations for conference papers and journal articles.

Conference papers. Use `@inproceedings` with `booktitle`, `series`, `volume`, `pages`, `year`, and `doi`. The following entry is printed as [DJKV17].

```
@inproceedings{dehnert2017storm,
  author    = {Dehnert, Christian and Junges, Sebastian
               and Katoen, Joost-Pieter and Volk, Matthias},
  title     = {A Storm is Coming: A Modern Probabilistic
               Model Checker},
  booktitle = cav # "~'17",
  series    = lncs,
  volume    = {10427},
  pages     = {592--600},
  year      = {2017},
  doi       = {10.1007/978-3-319-63390-9_31}
}
```

Journal articles. Use `@article` with `journal`, `volume`, `number` (if any), `pages`, `year`, and `doi`. The following entry is printed as [HJK+22].

```
@article{hensel2022storm,
  author  = {Hensel, Christian and Junges, Sebastian
             and Katoen, Joost-Pieter and Quatmann, Tim
             and Volk, Matthias},
  title   = {The Probabilistic Model Checker Storm},
  journal = sttt,
  volume  = {24},
  number  = {4},
  pages   = {589--610},
  year    = {2022},
  doi     = {10.1007/s10009-021-00633-z}
}
```

Books. Use `@book` with `publisher`, `year`, and `doi`, and if applicable `edition` or `series` and `volume`. For an edited book, give the `editor` instead of the `author`. Cite a specific page or chapter with the optional argument of `\cite`, e.g., [BK08, p. 768]. The following entry is printed as [Hig20].

```
@book{higham2020handbook,
  author    = {Higham, Nicholas J.},
  title     = {Handbook of Writing for the Mathematical
               Sciences},
  edition   = {Third},
  publisher = {SIAM},
  year      = {2020},
  doi       = {10.1137/1.9781611976106}
}
```

Book chapters. For a chapter of an edited book, where each chapter may have a different author, use `@incollection` with `booktitle`, `editor`, `pages`, `publisher`,

`year`, and `doi`. The following entry is printed as [BdFK18].

```
@incollection{baier2018model,
  author    = {Baier, Christel and {de Alfaro}, Luca
               and Forejt, Vojt{\v{e}}ch and Kwiatkowska, Marta},
  title     = {Model Checking Probabilistic Systems},
  booktitle = {Handbook of Model Checking},
  editor    = {Clarke, Edmund M. and Henzinger, Thomas A.
               and Veith, Helmut and Bloem, Roderick},
  pages     = {963--999},
  publisher = springer,
  year      = {2018},
  doi       = {10.1007/978-3-319-10575-8_28}
}
```

Theses. Use `@mastersthesis` or `@phdthesis` with the university in `school`. For a bachelor thesis, add `type = {Bachelor thesis}` to a `@mastersthesis` entry. The following entries are printed as [Mic25] and [Qua23].

```
@mastersthesis{michnia_thesis:2026,
  author = {Michnia, Jan-Gustav},
  title  = {Probabilistic Model Checking for Compliance
            Checking and Decision Point Optimization
            in Business Processes},
  school = {RWTH Aachen University},
  year   = {2026}
}

@phdthesis{quatmann_diss:2023,
  author = {Quatmann, Tim},
  title  = {Verification of Multi-Objective Markov Models},
  school = {RWTH Aachen University},
  year   = {2023}
}
```

Technical reports. Use `@techreport` with `institution` and `number`. The style prints "Technical Report" before the number, so the number in the bibliography entry should omit these words (use `type` to print other words, e.g., `type = {Preprint}`).

Websites and software. Use `@misc` with the author, the title, and the year of the cited version. Put the URL into `howpublished = {\url{...}}` and the access date into `note`, e.g., [Kas26].

DOIs

Add the DOI of every source that has one, as the bare identifier without `https://doi.org/`, e.g., [Lam12, Zob14]. Never put a URL into the `doi` field.

Appendix C

Mathematical Writing

The guidelines below summarize how to write a clear mathematical thesis. Delete this appendix before you submit your thesis.

The guidelines are taken from classic sources on mathematical and scientific writing, which are worth reading in full: Halmos [Hal70], Knuth et al. [KLR89], Gopen and Swan [GS90], Lamport [Lam12], Zobel [Zob14], and Higham [Hig20]. The guiding principle behind all of them is that you write for the *reader*: a fellow student who knows the basics of computer science but not your topic. Everything that makes the reader stop, search, or guess is a problem of the text, not of the reader.

C.1 Structure and Flow

- **Intuition first, then formality, then an example.** Before each definition or theorem, say in words what it formalizes and why it is needed. After it, illustrate it with an example. Use one running example throughout a chapter instead of many unrelated ones.
- **One topic per paragraph.** The first sentence of a paragraph states its topic, the remaining sentences develop it.
- **Old information first, new information last.** Start a sentence with what the reader already knows and put the new or important part at the end, where the reader expects the emphasis. Keep the subject and its verb close together.
- **Signposting.** Begin every chapter by saying what it contains and end it by summarizing what was achieved and where it is used next. Within long sections, tell the reader where the argument is going ("To prove Theorem 3.6, we first show an auxiliary lemma.").
- **Present only what you need.** The preliminaries contain exactly the notions used later. Do not include definitions or results "for completeness" that play no

role in your thesis.

C.2 Language and Style

- **Use "we"** consistently, meaning the author and the reader ("We now show that ..."). Never use "I"!
- **Prefer the active voice and the present tense** ("Theorem 3.6 shows ..."). Use the past tense only for things that happened at a specific time, e.g., when you describe the experiments you ran.
- **Be precise.** Replace vague words like "very", "quite", "some", or "a lot" by concrete statements or numbers ("solves 23 more benchmarks" instead of "solves many more benchmarks"). Avoid ambiguous "it" and "this" and name the object instead.
- **Abbreviations** are spelled out at their first use, e.g., "discrete-time Markov chain (DTMC)", and are not used in the abstract.
- **Spelling.** Use either American or British English consistently, and run a spell checker before submission. Write "e.g.," and "i.e.," with a comma afterwards.

C.3 Definitions and Notation

- **Define everything before you use it,** and define it only once. If a notion was defined many pages before, recall it briefly and refer to the definition ("Recall that a state s is *absorbing* if $\mathbf{P}(s, s) = 1$ (see Definition 3.4).").
- **Emphasize the defined term** where it is defined, e.g., "A program is called *almost-surely terminating* if ...".
- **Definitions use "if" and "=".** In a definition, "if" already means "if and only if", so never write "iff" or "if and only if" in a definition. Likewise, define objects with "=" ("Let $f(x) = x + 1$."), not with `:=`, `\coloneqq`, or `::=`. Use "iff" or "if and only if" only in statements that you prove, e.g., in a theorem or lemma, where both directions have to be shown.
- **One symbol, one meaning.** Never use the same symbol for two different objects, and never use two symbols for the same object. Keep the conventions consistent: e.g., lowercase letters for elements, uppercase letters for sets, calligraphic letters for systems.
- **Do not introduce notation that is used only once.** Every new symbol is a burden for the reader; a few words are often clearer.
- **One word, one concept.** In technical writing, varying the words for the same concept (state, configuration, location) suggests that they mean different things.

Pick one word and stick to it.

- **Use standard notation** wherever it exists, e.g., the notation of the standard textbook of your area such as [BK08]. If your thesis uses many symbols, add a list of notation to the front matter.

C.4 Formulas in the Text

Mathematical formulas are part of the sentences they appear in and follow the same rules of grammar and punctuation.

- **Do not start a sentence with a symbol.**

- **Do not separate two formulas by only a punctuation mark**; put words in between.

- **Do not use logical symbols as abbreviations in prose.** Symbols like $\forall$, $\exists$, $\Rightarrow$, or $\wedge$ belong inside formulas, not between words.

- **Punctuate displayed formulas.** A displayed formula ends with a comma or a period if the sentence does, e.g.,

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1, \tag{C.1}$$

  where $n \in \mathbb{N}$.

- **Numbering.** Use `\begin{equation}` with a `\label` for displayed formulas that you refer to, e.g., `\cref{eq:mw-geometric-sum}` yields Equation (C.1). Use `\begin{equation*}` for formulas that are never referenced.

- **Words in formulas** are set with `\text`, and operator names with `\operatorname` or the predefined commands such as `\max` and `\log`, so that they are not typeset as products of variables: $\max(x, y)$ instead of $max(x, y)$.

- **Use the right symbols:** `\colon` for the type of a function ($f \colon \mathbb{N} \to \mathbb{N}$), `\mid` in set comprehensions ($\{x \in \mathbb{N} \mid x > 0\}$), `\ldots` between commas ($x_1, \ldots, x_n$), and `\cdots` between operators ($x_1 + \cdots + x_n$).

Table C.1 shows typical mistakes and how to avoid them.

Table C.1: Typical mistakes in mathematical prose.

Avoid	**Prefer**
x is positive, so …	The value x is positive, so …
Since $x > 0$, $y < 0$.	Since $x > 0$, we have $y < 0$.
$\forall s \in S$ we have $\mathbf{P}(s, s) = 1$.	For all $s \in S$, we have $\mathbf{P}(s, s) = 1$.
If s is absorbing $\Rightarrow$ s cannot be left.	If s is absorbing, then s cannot be left.
As shown in Theorem 3.6, …	As shown in Theorem 3.6, …
This follows from it.	This lemma follows from Theorem 3.6.
Obviously, the program terminates almost surely.	The program terminates almost surely, since the loop is exited with probability $1/2$ in every iteration.
(Definition) A state is *absorbing* iff …	(Definition) A state is *absorbing* if …
(Definition) Let $f(x) \coloneqq x + 1$.	(Definition) Let $f(x) = x + 1$.

C.5 Theorems and Proofs

- **Self-contained statements.** A theorem lists all of its assumptions. Avoid statements like "Under the assumptions above, …", since the reader has to search for what "above" means.
- **Explain the significance** of every result: why it is needed, how it is used later, and what it means intuitively.
- **Announce the structure of a proof** at its beginning ("We prove the claim by induction on n.", "Assume for a contradiction that …"). Mark every case clearly and state the induction hypothesis explicitly.
- **Justify every step** and refer to the definition or result it uses by its label. Structure long proofs hierarchically into numbered steps, each with its own short justification.
- **Avoid "obviously", "clearly", and "trivially".** If a step is obvious, these words are unnecessary; if it is not, they hide a gap. Give the reason instead.
- **Long proofs go to the appendix.** Give a proof sketch in the main text and move the full proof to Appendix A.
- **Apply the result to an example** after the proof, ideally to the running example.

Lemma 3.5 and its proof in Section 3.3 illustrate these guidelines. The sentence before the lemma explains why it is needed, and the proof announces its structure, marks the

base case and the induction step, states the induction hypothesis, and justifies every equality. The full proof of Theorem 3.6 in Appendix A.1 shows how to structure a longer proof into numbered steps.

C.6 Figures and Tables

- **Every figure and table is referenced** in the text with `\cref`, and the text explains what the reader should see in it.
- **Captions are self-contained**: a reader who only looks at the figure and its caption should understand what is shown, including the meaning of all abbreviations and units.
- **Tables** use horizontal rules only (`\toprule`, `\midrule`, `\bottomrule` from `booktabs`) and no vertical rules, see Tables C.1 and 4.1.
- **Colors** must not be the only way to distinguish information: the thesis has to be readable in black and white and by readers with color vision deficiency.